# MATHEMATICS

***(Maximum Marks: 100)***

***(Time allowed: Three hours)***

*(Candidates are allowed additional 15 minutes for* ***only*** *reading the paper.*

*They must NOT start writing during this time.)*

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*The Question Paper consists of three sections A, B and C.*

*Candidates are required to attempt all questions from* ***Section A*** *and all questions*

***EITHER*** *from* ***Section B OR Section C***

***Section A:*** *Internal choice has been provided in three questions of four marks each and two questions of six marks each.*

***Section B:*** *Internal choice has been provided in two questions of four marks each.*

***Section C:*** *Internal choice has been provided in two questions of four marks each.*

*All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.*

*The intended marks for questions or parts of questions are given in brackets [ ].*

***Mathematical tables and graph papers are provided.***

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## SECTION A (80 Marks)

**Question 1** **[10×2]**

(i) If $f$ : R→R, $f(x) = x^3$ and $g$ : R→ R, $g(x) = 2x^2+1$, and R is the set of real numbers, then find $fog\,(x)$ and $gof\,(x)$.

(ii) Solve: $\text{Sin}\,(2\tan^{-1}x) = 1$

(iii) Using determinants, find the values of k, if the area of triangle with vertices (−2, 0), (0, 4) and (0, k) is 4 square units.

(iv) Show that (A + A′) is symmetric matrix, if $A = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$.

(v) $f(x) = \dfrac{x^2 - 9}{x - 3}$ is not defined at $x = 3$. What value should be assigned to $f(3)$ for continuity of $f(x)$ at $x = 3$?

(vi) Prove that the function $f(x) = x^3 - 6x^2 + 12x + 5$ is increasing on R.

(vii) Evaluate: $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$

(viii) Using L'Hospital's Rule, evaluate: $\lim_{x \to 0} \frac{8^x - 4^x}{4x}$

(ix) Two balls are drawn from an urn containing 3 white, 5 red and 2 black balls, one by one without replacement. What is the probability that at least one ball is red?

(x) If events A and B are independent, such that $P(A) = \frac{3}{5}$, $P(B) = \frac{2}{3}$, find $P(A \cup B)$.

**Question 2** **[4]**

If $f: A \to A$ and $A = R - \left\{ \frac{8}{5} \right\}$, show that the function $f(x) = \frac{8x+3}{5x-8}$ is one – one onto. Hence, find $f^{-1}$.

**Question 3** **[4]**

(a) Solve for $x$:

$$\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$$

**OR**

(b) If $\sec^{-1} x = \operatorname{cosec}^{-1} y$, show that $\frac{1}{x^2} + \frac{1}{y^2} = 1$

**Question 4** **[4]**

Using properties of determinants prove that:

$$\begin{vmatrix} x & x(x^2+1) & x+1 \\ y & y(y^2+1) & y+1 \\ z & z(z^2+1) & z+1 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

**Question 5** **[4]**

(a) Show that the function $f(x) = |x-4|,\ x \in R$ is continuous, but not differentiable at $x = 4$.

**OR**

(b) Verify the Lagrange's mean value theorem for the function:

$$f(x) = x + \frac{1}{x} \text{ in the interval } [1, 3]$$

**Question 6** **[4]**

If $y = e^{\sin^{-1}x}$ and $z = e^{-\cos^{-1}x}$, prove that $\frac{dy}{dz} = e^{\pi/2}$

**Question 7** **[4]**

A 13 m long ladder is leaning against a wall, touching the wall at a certain height from the ground level. The bottom of the ladder is pulled away from the wall, along the ground, at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall?

**Question 8** **[4]**

(a) Evaluate: $\int \frac{x(1+x^2)}{1+x^4} dx$

**OR**

(b) Evaluate: $\int_{-6}^{3} |x+3| dx$

**Question 9** **[4]**

Solve the differential equation: $\frac{dy}{dx} = \frac{x+y+2}{2(x+y)-1}$

**Question 10** **[4]**

Bag A contains 4 white balls and 3 black balls, while Bag B contains 3 white balls and 5 black balls. Two balls are drawn from Bag A and placed in Bag B. Then, what is the probability of drawing a white ball from Bag B?

**Question 11** **[6]**

Solve the following system of linear equations using matrix method:

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=9$$

$$\frac{2}{x}+\frac{5}{y}+\frac{7}{z}=52$$

$$\frac{2}{x}+\frac{1}{y}-\frac{1}{z}=0$$

**Question 12** **[6]**

(a) The volume of a closed rectangular metal box with a square base is 4096 $cm^3$. The cost of polishing the outer surface of the box is ₹ 4 per $cm^2$. Find the dimensions of the box for the minimum cost of polishing it.

**OR**

(b) Find the point on the straight line $2x + 3y = 6$, which is closest to the origin.

**Question 13** **[6]**

Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

**Question 14** **[6]**

(a) Given three identical Boxes A, B and C, Box A contains 2 gold and 1 silver coin, Box B contains 1 gold and 2 silver coins and Box C contains 3 silver coins. A person chooses a Box at random and takes out a coin. If the coin drawn is of silver, find the probability that it has been drawn from the Box which has the remaining two coins also of silver.

**OR**

(b) Determine the binomial distribution where mean is 9 and standard deviation is $\frac{3}{2}$ .

Also, find the probability of obtaining at most one success.

## SECTION B (20 Marks)

**Question 15** **[3×2]**

(a) If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a}+\vec{b}|=13$ and $|\vec{a}|=5$, find the value of $|\vec{b}|$.

(b) Find the length of the perpendicular from origin to the plane $\vec{r}.(3i-4j-12k)+39=0$.

(c) Find the angle between the two lines $2x = 3y = -z$ and $6x = -y = -4z$ .

**Question 16** **[4]**

(a) If $\vec{a}=i-2j+3k,\ \vec{b}=2i+3j-5k$, prove that $\vec{a}$ and $\vec{a}\times\vec{b}$ are perpendicular.

**OR**

(b) If $\vec{a}$ and $\vec{b}$ are non-collinear vectors, find the value of $x$ such that the vectors $\vec{\alpha}=(x-2)\vec{a}+\vec{b}$ and $\vec{\beta}=(3+2x)\vec{a}-2\vec{b}$ are collinear.

**Question 17** **[4]**

(a) Find the equation of the plane passing through the intersection of the planes $2x + 2y - 3z - 7 = 0$ and $2x + 5y + 3z - 9 = 0$ such that the intercepts made by the resulting plane on the $x$-axis and the $z$-axis are equal.

**OR**

(b) Find the equation of the lines passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$

**Question 18** **[6]**

Draw a rough sketch and find the area bounded by the curve $x^2 = y$ and $x + y =2$.

## SECTION C (20 Marks)

**Question 19** **[3×2]**

(a) A company produces a commodity with ₹ 24,000 as fixed cost. The variable cost estimated to be 25% of the total revenue received on selling the product, is at the rate of ₹ 8 per unit. Find the break-even point.

(b) The total cost function for a production is given by $C(x)=\frac{3}{4}x^2-7x+27$.

Find the number of units produced for which M.C. = A.C.
(M.C.= Marginal Cost and A.C. = Average Cost.)

(c) If $\overline{x}=18, \overline{y}=100, \sigma_x=14, \sigma_y=20$ and correlation coefficient $r_{xy}=0\cdot 8$, find the regression equation of $y$ on $x$.

**Question 20** **[4]**

(a) The following results were obtained with respect to two variables $x$ and $y$:

$$\sum x=15, \sum y=25, \sum xy=83, \sum x^2=55, \sum y^2=135 \text{ and } n=5$$

(i) Find the regression coefficient $b_{xy}$.

(ii) Find the regression equation of $x$ on $y$.

**OR**

(b) Find the equation of the regression line of $y$ on $x$, if the observations $(x, y)$ are as follows:

(1, 4), (2, 8), (3, 2), (4, 12), (5, 10), (6, 14), (7, 16), (8, 6), (9, 18)

Also, find the estimated value of $y$ when $x = 14$.

**Question 21** **[4]**

(a) The cost function of a product is given by $C(x)=\frac{x^3}{3}-45x^2-900x+36$ where $x$ is the number of units produced. How many units should be produced to minimise the marginal cost?

**OR**

(b) The marginal cost function of $x$ units of a product is given by MC = $3x^2-10x+3$. The cost of producing one unit is ₹ 7. Find the total cost function and average cost function.

**Question 22** **[6]**

A carpenter has 90, 80 and 50 running feet respectively of teak wood, plywood and rosewood which is used to produce product A and product B. Each unit of product A requires 2, 1 and 1 running feet and each unit of product B requires 1, 2 and 1 running feet of teak wood, plywood and rosewood respectively. If product A is sold for ₹ 48 per unit and product B is sold for ₹ 40 per unit, how many units of product A and product B should be produced and sold by the carpenter, in order to obtain the maximum gross income?
Formulate the above as a Linear Programming Problem and solve it, indicating clearly the feasible region in the graph.